Modulational instability of short-wavelength ion waves in strongly coupled dusty plasmas

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Modulational instability of electrostatic short-wavelength ion waves in plasmas containing strongly coupled dusts with variable charge is considered. The evolution equations for the ion waves modulated by slow dust motion are obtained. The instability behavior differs considerably from that of plasmas with weakly coupled dust grains and depends strongly on the Coulomb coupling parameter as well as the dust-charge relaxation rate.

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Several studies have been devoted to the modulation of ion waves by low-frequency dust motion such as that of dust acoustic waves and dust-modified ion acoustic waves [1–6]. Most of these investigations are for weakly coupled dust grains. Since in many laboratory situations the typical dust charge is large, the Coulomb coupling parameter $\Gamma = Z_d^2 e^2 / a_d T_d$ (where Z_d is the dust-charge number, $-e$ is the electronic charge, a_d is the average interdust distance, and T_d is the dust temperature) is often much larger than unity, the dust grains can be strongly correlated [7–13]. In this paper we investigate the modulational instability of short-wavelength ion waves [2,14] in plasmas containing strongly coupled dust grains modeled by the generalized viscoelastic hydrodynamic theory [7]. A set of Zakharov-like evolution equations is derived and analyzed. It is found that dust correlation as well as dust-charge relaxation can strongly affect the modulational process.

We assume that $r \ll \lambda_p$ and $a_d \sim$ or $\ll \lambda$, where r is the dust size, $\lambda_p = (\lambda_i^{-2} + \lambda_e^{-2})^{-1/2}$ is the plasma Debye length, which actually also depends on the dust density because of quasineutrality, a_d is the interdust distance, $\lambda_{i,e}$ are the ion and electron Debye lengths, and λ is the perturbation wavelength, or the spatial scale of dust motion. In the macroscopic picture of the waves the dust grains can, therefore, be treated as a negatively charged fluid with a (variable) charge density given by $n_d Q_d = -n_d Z_d e < 0$, where n_d is the dust density. We also assume $T_e \gg T_i \gg T_d$, $m_e \ll m_i \ll m_d / Z_d$, and that $n_{d0} Z_{d0} \sim n_{i0} \gg n_{e0}$, where m_α, $n_{\alpha 0}$, and T_α are the mass, steady-state number density, and temperature of species α $(=i,e,d)$, respectively. Thus, for motion on the electron or ion time scale, the plasma is non-neutral and wave regimes that are not accessible in neutral plasmas can become accessible.

Ion acoustic waves are subject to modulation by the very low-frequency dust motion. The dispersion relation for the ion acoustic waves is given by

$$\omega_{\mathbf{k}_0}^2 = \frac{\omega_{pi}^2 |\mathbf{k}_0|^2 \lambda_e^2}{1 + |\mathbf{k}_0|^2 \lambda_e^2}, \tag{1}$$

where $\omega_{\mathbf{k}_0}$ and $\mathbf{k}_0$ are the frequency and wave vector of the ion waves, and ω_{pi} is the ion plasma frequency. In the short-wavelength regime,

$$|\mathbf{k}_0| \lambda_i \ll 1 \ll |\mathbf{k}_0| \lambda_e < \sqrt{m_i / m_e}, \tag{2}$$

where the last inequality precludes strong Landau damping [2], the frequency of the ion waves can be written as

$$\omega_{\mathbf{k}_0} = \omega_{pi} \left(1 - \frac{1}{2 |\mathbf{k}_0|^2 \lambda^2 e} \right), \tag{3}$$

which is sometimes called the ion Langmuir wave frequency.

It has been shown [2] that short-wavelength ion acoustic waves are modulated by weakly coupled ($\Gamma < 1$) dust grains, and the resulting instability should appear in dusty plasmas. The modulation can be by low-frequency dust acoustic waves with wavelength $\lambda (= 2\pi / |\mathbf{k}|)$ larger as well as smaller than that of the pump ion waves. For weakly coupled dust grains the dispersion relation of slow dust acoustic waves is

$$\omega_{\mathbf{k}}^2 \approx |\mathbf{k}|^2 v_{sd}^2, \tag{4}$$

where

$$v_{sd}^2 = \omega_{pd}^2 \lambda_i^2 = \frac{Z_{d0}^2 n_{d0} T_i}{n_{i0} m_d} \tag{5}$$

is the dust acoustic speed and $\omega_{pd} = \sqrt{4\pi n_{d0} Z_{d0}^2 e^2 / m_d}$ is the dust plasma frequency. For strongly coupled ($\Gamma > 1$) dust grains, however, the dust response can be quite different because of the significant difference in the dispersive properties of the low-frequency dust motion for weakly and strongly coupled dust grains [11,13].

From the generalized viscoelastic hydrodynamic equations [7] for the dust grains one can obtain [13] the dispersion relation for the dust waves:

$$\frac{\omega^2}{\omega_{pd}^2} + \frac{\mathrm{i}(\omega/\omega_{pd}) k^2 a_d^2 \eta'}{1 - \mathrm{i}\omega \tau_m} = \gamma_d \mu_d k^2 \lambda_d^2 + \frac{k^2 a_d^2}{\mathcal{B}\kappa^2 + k^2 a_d^2}, \tag{6}$$

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where τ_m is the viscoelastic energy-relaxation time, $\eta' = (\xi + 4\eta/3)/m_d n_{d0}\omega_{pd} a_d^2$ is the normalized viscosity, ξ and η are the bulk and shear viscosities, γ_d is the adiabatic index, μ_d is the compressibility, λ_d is the Debye length of the dust grains, and $\kappa = a_d/\lambda_p \approx a_d/\lambda_i$.

Using the probe model [8,15] for dust charging we have

$$\mathcal{B} = I + rP\omega_{pi}\tau_d^{cb}/\sqrt{2\pi}\lambda_i, \tag{7}$$

where $P = Z_{d0}n_{d0}/n_{e0} \sim O(10)$ and $\tau_d^{\mathrm{ch}} \sim \sqrt{2\pi}\lambda_i/r\omega_{pi}(1 + Z_{d0}e^2/rT_e)$ is the dust-charge relaxation time [8]. The model is valid for stationary perfectly conducting spherical dust grains and $a_d \ll \lambda$, but is often applied to dust grains in general [3–6]. For weakly coupled constant-charge dust grains, Eq. (4) can be recovered from Eq. (6) if we set $\eta' = 0$, $\tau_m = 0$, $\gamma_d\mu_d = 1$, $\mathcal{B} = 1$, and $k\lambda_i \ll 1$. For strongly coupled variable-charge dust grains, Eq. (6) can be rewritten as

$$\omega_{\mathbf{k}}^2 = |\mathbf{k}|^2 v_{sd1}^2, \tag{8}$$

where $v_{sd1}^2 = \mathcal{C}_1 v_{sd}^2$ and $\mathcal{C}_1 = \mathcal{B}^{-1} + \kappa^2(1 - 0.24\Gamma)/3\Gamma$. The condition for Eq. (8) to hold is $k^2a_d^2 \ll \mathcal{B}\kappa^2$. Satisfying this condition are the long-wavelength ($ka_d \ll 1$), weak dust charging ($\mathcal{B}\kappa^2 \sim 1$) regimes for moderately strongly coupled dust grains ($\Gamma \sim 1$), and the relatively short-wavelength ($ka_d \sim 1$), strong dust charging ($\mathcal{B}\kappa^2 \gg 1$) regime for highly strongly coupled dust grains ($\Gamma \gg 1$). For propagating waves we have $\mathcal{C}_1 > 0$, so that $\kappa < \kappa_c = \sqrt{3\Gamma/\mathcal{B}(0.24\Gamma - 1)}$ when $\Gamma > 1/0.24$. In general, it is always valid since $\kappa \sim 1$ or $\ll 1$, for example, $\kappa_c \approx 3$ when given $\Gamma \sim 100$ and $\mathcal{B} \sim 1$. From Eq. (8) one can also see that the value of $\mathcal{C}_1$ is very small for strong charge relaxation and strong dust coupling, demonstrating the existence of slow dust acoustic waves with ultralow phase velocity v_{sd1} and large wave number $|\mathbf{k}| \gg |\mathbf{k}_0|$. In this case, for $a_d \sim \lambda_p \sim \lambda_i \ll \lambda_e$ and $ka_d \sim 1$, we have $|\mathbf{k}_0|\lambda_i \ll 1$, which is consistent with Eq. (2). It should be noted that the terms short and long wavelengths have different meanings when applied to the pump (the ion waves) and the modulation (the dust motion).

The derivation of the equations governing wave modulation is standard [16]. First, one notes that the ion waves are of much higher frequency than that of the dust waves. The dynamics of each of these waves are treated as linear within their own time scales. The nonlinear coupling of the ion waves is through the modulation of the background (originally constant) ion density by the slow dust motion, and that of the dust motion is induced by the slowly varying ponderomotive force exerted on the dust grains by the ion waves. The electric field $\mathbf{E}$ of the short-wavelength ion waves can be written as

$$\mathbf{E} = \tfrac{1}{2}[\mathbf{E}_{iL}\exp -\mathrm{i}\omega_{pi}t) + \mathrm{c.c.}], \tag{9}$$

where $\mathbf{E}_{iL}(\mathbf{r},t)$ is the slowly varying electric-field envelope arising from density modulation by the dust grains. Accordingly we have

$$\left(i\frac{\partial}{\partial t}\Delta - \frac{\omega_{pi}}{2\lambda_e^2}\right)\nabla\cdot\mathbf{E}_{iL} = \frac{\omega_{pi}}{2}\nabla\cdot\Delta\left(\frac{\delta n_i}{n_{i0}}\mathbf{E}_{iL}\right), \tag{10}$$

where Δ is the Laplacian and δn_i is the ion density modulation. The left-hand side if set to zero is the linear ion wave equation and the nonlinear right-hand side is due to a modulation of the equilibrium ion density (or the ion plasma frequency). We note that the coupling of long-wavelength ion waves to the dust motion is much weaker [2] since such ion waves are almost nondispersive.

From Eq. (8) the density modulation is governed by

$$\left(\frac{\partial^2}{\partial t^2} - v_{sd1}^2\Delta\right)\frac{\delta n_i}{n_{i0}} = \frac{Z_{d0}^2 n_{d0}}{16\pi n_{i0}^2 m_d}\Delta|\mathbf{E}_{iL}|^2, \tag{11}$$

where the left-hand side describes dust acoustic waves and the right-hand side is the ponderomotive force exerted on the dust grains by the ion waves. Equation (11) is valid for $|\mathbf{k}| \ll |\mathbf{k}_0|$, as well as for $|\mathbf{k}| \gg |\mathbf{k}_0|$ when charging and coupling of the dust grains are strong enough such that $\mathcal{B}\kappa^2 \gg k^2a_d^2$ holds [see also the discussion on the validity of Eq. (8)]. For example, if $P \sim 20$, $\Gamma \sim 10$, $|\mathbf{k}|a_d \approx 1$, and $a_d \sim \lambda_i$, we have $\mathcal{B} \sim 10$ and $\kappa \sim 1$. Thus the conditions $\mathcal{B}\kappa^2 \gg k^2a_d^2$ and $|\mathbf{k}| \gg |\mathbf{k}_0|$ are both satisfied.

Equations (10) and (11) constitute a set of coupled partial differential equations describing the modulation of the ion Langmuir waves by low-frequency dust motion. For a nearly monochromatic short-wavelength ion wave, the wave envelope modulated by low-frequency dust motion $(\omega_{\mathbf{k}},\mathbf{k})$ can be written as

$$\mathbf{E}_{iL} = \mathbf{E}_0\exp[-i(\omega_{\mathbf{k}}t - \mathbf{k}\cdot\mathbf{x})],$$

which leads to the appearance of two sidebands $(\omega \pm \omega_{\mathbf{k}_0}, \mathbf{k} \pm \mathbf{k}_0)$ of the short-wavelength ion waves. The corresponding nonlinear dispersion relation for modulational interaction is then [2]

$$\left\{\left[\frac{\mathbf{k}_0\cdot(\mathbf{k}+\mathbf{k}_0)}{|\mathbf{k}_0||\mathbf{k}+\mathbf{k}_0|}\right]^2\frac{1}{\mathcal{D}_+} + \left[\frac{(\mathbf{k}_0\cdot\mathbf{k}_0)}{|\mathbf{k}_0||\mathbf{k}-\mathbf{k}_0|}\right]^2\frac{1}{\mathcal{D}_-}\right\}\beta|\mathbf{E}_0|^2 = 1, \tag{12}$$

where the linear dielectric permittivities of the sidebands in the general form are

$$\mathcal{D}_\pm = 1 - \frac{\omega_{pi}^2}{(\omega \pm \omega_{\mathbf{k}_0})^2} + \frac{1}{|\mathbf{k}+\mathbf{k}_0|^2\lambda_e^2}, \tag{13}$$

where the coupling parameter β is given by

$$\beta = \frac{1}{32\pi n_{i0}T_i}\frac{|\mathbf{k}|^2 v_{sd}^2}{\omega^2 - |\mathbf{k}|^2 v_{sd1}^2}, \tag{14}$$

which governs the coupling of the ion waves to the very low-frequency dust motion.

The dielectric permittivities can be simplified. Accordingly, for $|\mathbf{k}| \ll |\mathbf{k}_0|$, we have

$$\left[\frac{\mathbf{k}_0\cdot(\mathbf{k}\pm\mathbf{k}_0)}{|\mathbf{k}_0||\mathbf{k}\pm\mathbf{k}_0|}\right]^2\sim 1-\left(\frac{|\mathbf{k}|^2}{|\mathbf{k}_0|^2}\sin^2\theta\right)\left(1\pm\frac{2|\mathbf{k}|}{|\mathbf{k}_0|}\cos\theta\right)^{-1} \tag{15}$$

and

$$\mathcal{D}_\pm\sim\pm 2\left[\left(1+\frac{1}{|\mathbf{k}_0|^2\lambda_e^2}\right)\frac{\omega}{\omega_{pi}}-\frac{|\mathbf{k}|\cos\theta}{|\mathbf{k}_0|^3\lambda_e^2}\right], \tag{16}$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{k}_0$. On the other hand, we have

$$\left[\frac{\mathbf{k}_0\cdot(\mathbf{k}\pm\mathbf{k}_0)}{|\mathbf{k}_0||\mathbf{k}\pm\mathbf{k}_0|}\right]^2\sim\cos^2\theta\pm\frac{2|\mathbf{k}_0|^2}{|\mathbf{k}|^2}\cos\theta\sin^2\theta \tag{17}$$

and

$$\mathcal{D}_\pm\sim-\frac{1}{|\mathbf{k}_0|^2\lambda_e^2}\pm 2\left(1+\frac{1}{|\mathbf{k}_0|^2\lambda_e^2}\right)\frac{\omega}{\omega_{pi}} \tag{18}$$

for $|\mathbf{k}|\gg|\mathbf{k}_0|$.

In order to compare the present results with that for weakly coupled dust grains, we shall now consider several limiting cases. Substituting Eqs. (15) and (16) into Eq. (12) one obtains

$$-\frac{2\omega_{pi}|\mathbf{k}|^3\sin^2\theta\cos\theta}{|\mathbf{k}_0|^3(1+|\mathbf{k}_0|^{-2}\lambda_e^{-2})\omega}\beta|\mathbf{E}_0|^2=1 \tag{19}$$

for the dispersion relation of long-wavelength ($|\mathbf{k}|\ll|\mathbf{k}_0|$) modulation. For $\gamma\gg kv_{sd1}$, we obtain the growth rate

$$\gamma\sim\left[\frac{v_{sd}^2\omega_{pi}\mathcal{W}|\mathbf{k}|^5\sin^2\theta\cos\theta}{|\mathbf{k}_0|^3(1+|\mathbf{k}_0|^{-2}\lambda_e^{-2})}\right]^{1/3}, \tag{20}$$

where $\mathcal{W}=|\mathbf{E}_0|^2/16\pi n_{i0}T_i$. For $\cos\theta=1/\sqrt{3}$ it corresponds to the interaction with the largest growth rate

$$\gamma_{\max}\sim\left[\frac{|\mathbf{k}|^5 v_{sd}^2\omega_{pi}\mathcal{W}}{|\mathbf{k}_0|^3(1+|\mathbf{k}_0|^{-2}\lambda_e^{-2})}\right]^{1/3}, \tag{21}$$

so that the growth rate is similar to that for weakly coupled dust grains [see Eq. (38) of [2]]. However, compared to the case of weakly coupled dust grains, the threshold of the instability is now significantly lowered. This is because the condition $\gamma\gg kv_{sd1}$ can be easily satisfied since usually $v_{sd1}\ll v_{sd}$, while the condition $\gamma\gg kv_{sd}$ is difficult to satisfy because of the large value of v_{sd} for a typical dusty plasma. For example, given $P=20$, $\Gamma=10$, and $\kappa\approx 1$, we have $\mathcal{B}\approx 10$ and $v_{sd1}\sim 0.2v_{sd}$.

Substituting Eqs. (17) and (18) into Eq. (12), we obtain for the dispersion relation in the regime $|\mathbf{k}|\gg|\mathbf{k}_0|$:

$$\frac{\omega_{pi}^2\cos^2\theta}{2|\mathbf{k}_0|^2\lambda_e^2[\omega^2-(\omega_{pi}|\mathbf{k}_0|^{-2}\lambda_e^{-2})^2]}\beta|\mathbf{E}_0|^2=1, \tag{22}$$

so that the maximum growth rate corresponds to $\mathbf{k}\|\mathbf{k}_0$.

The growth rate can be obtained from Eq. (22). It scales differently in different parameter regimes. First, for $\omega_{pi}/|\mathbf{k}_0|^2\lambda_e^2\gg\omega\gg kv_{sd_1}$, we obtain

$$\gamma_{\max}\sim\sqrt{|\mathbf{k}_0|^2|\mathbf{k}|^2\lambda_e^2 v_{sd}^2\mathcal{W}}, \tag{23}$$

so that the growth rate increases with k. Next, for $\omega\gg\max[kv_{sd1},\omega_{pi}/|\mathbf{k}_0|^2\lambda_e^2]$, we obtain

$$\gamma_{\max}\sim\left(\frac{|\mathbf{k}|^2 v_{sd}^2\omega_{pi}^2\mathcal{W}}{4|\mathbf{k}_0|^2\lambda_e^2}\right)^{1/4}, \tag{24}$$

which increases with $\sqrt{k}$. In both of the above cases the growth rates [Eqs. (23) and (24)] are independent of Γ and τ_d^{ch}. In other words, the growth rates are formally the same as that for weakly coupled dust grains. This is because for large k the parameter η' rapidly decreases [13]. In these two cases the instability has a lower threshold than the corresponding cases of weakly coupled plasmas because of dust coupling and charging also make $\gamma\gg kv_{sd1}$ easily satisfied since usually $v_{sd1}\ll v_{sd}$. Finally, for $\omega_{pi}/|\mathbf{k}_0|^2\lambda_e^2\ll\omega\ll kv_{sd1}$, we obtain the growth rate

$$\gamma_{\max}\sim\left(\frac{\omega_{pi}^2\mathcal{W}}{4\mathcal{C}_1|\mathbf{k}_0|^2\lambda_e^2}\right)^{1/2}, \tag{25}$$

which is independent of k. However, the growth rate is now strongly dependent on the parameter $\mathcal{C}_1$ associated with dust coupling and charging. Since usually $v_{sd1}\ll v_{sd}$ (or $\mathcal{C}_1\ll 1$), the threshold is lower than that for weakly coupled dust grains.

In conclusion, we have considered the modulational instability of short-wavelength ion waves in plasmas containing strongly coupled dust grains. Dust-charge relaxation is taken into account. It is found that the modulation of the ion waves depends strongly on the Coulomb coupling parameter Γ, the dust-charge relaxation time τ_d^{ch}, as well as the ratio of average interdust distance to plasma Debye length κ (appearing in the parameter $\mathcal{C}_1$, or v_{sd1}, and the inequality $\gamma\gg kv_{sd1}$).

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